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Travel Grants Available for ICME-6

Mathematics educators, including pre-college classroom teachers, may qualify for travel grants for the Sixth International Congress on Mathematical Education. The Congress takes place from July 27 - August 3, 1988 in Budapest, Hungary. Information and applications for travel grants can be obtained by writing:

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 1906 Association Drive
 Reston, VA 22091

Applications must be received by March 1, 1988.

Answers

Solutions to the Quickies which appear on p. 330.

A726.

$$\begin{aligned}\lim_{a \rightarrow 1^-} \left[(1-a) \sum_{k=0}^{\infty} a^{ke} e^{a^{ke}} \right] &= \lim_{a \rightarrow 1^-} \left[\left(\frac{1-a}{1-a^e} \right) \sum_{k=0}^{\infty} (1-a^e) a^{ke} e^{a^{ke}} \right] \\ &= \left(\lim_{a \rightarrow 1^-} \frac{1-a}{1-a^e} \right) \left(\lim_{a \rightarrow 1^-} \sum_{k=0}^{\infty} (a^{ke} - a^{(k+1)e}) e^{a^{ke}} \right) \\ &= \frac{1}{e} \int_0^1 e^x dx = \frac{e-1}{e},\end{aligned}$$

since $\{a^{ke} | k=0, 1, 2, \dots\}$ is a partition of $(0, 1]$ and

$$|a^{ke} - a^{(k+1)e}| \leq 1 - a \rightarrow 0 \quad \text{as } a \rightarrow 1^-.$$

A727. Let C and $C(t)$ be the fixed circle and the rolling circle at time t , respectively. For a fixed time t , there is a unique sphere which contains C and $C(t)$ on its surface. (The center of this sphere is the intersection of the lines through the centers, and perpendicular to the planes, of C and $C(t)$.) This sphere is independent of t because θ is constant for all t . Thus, all points of $C(t)$, for all t , are on this sphere, and in particular, the curve traced by P is on this sphere.

